GURU NANAK COLLEGE, SRI MUKTSAR SAHIB

APPLICATIONS OF DERIVATIVES IN

UNDERSTANDING THE NATUTRE OF CURVES

A PROJECT REPORT

SUBMITTED UNDER

STAR COLLEGE SCHEME

DBT, GOVT. OF INDIA.

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ACKNOWLEDGEMENT

It is proud of my privilege and pleasure to express my deep sense of gratitude to my project guide, Ms. Kamaljeet Kaur (Assistant Professor, P.G. Department of Mathematics, Guru Nanak College, Sri Muktsar Sahib, Punjab.) for her help, valuable guidance and constant encouragement. I am also thankful to all teaching staff of P.G. Department of Mathematics who have rendered their invaluable guidance and assistance time to time.

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1. ABSTRACT

The derivative, in mathematics, is defined as the rate of change of a function with respect to a variable. Derivatives are fundamental to the solution of problems in calculus and differential equations. Applications of derivatives are applied in many circumstances like calculating the slope of curve, determining the maximaor minima of a function, obtaining the equation of a tangent and normal to a curve, concavity and convexity of curves along with their points of inflexion and multiple points etc.

2. INTRODUCTION

To differentiate a function, we need to find its derivative function using the formula

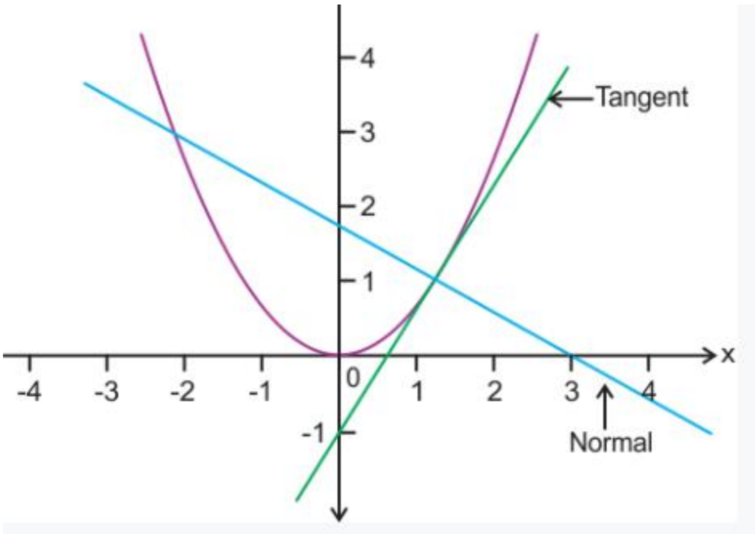
$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

We had studied about the computation of derivatives that is, how to find the derivatives of different function like composite functions, implicit functions, trigonometric functions, logarithmic functions and many more. But now in application of derivatives we will see how and where to apply the concept of derivatives. Generally the concept of derivatives are applied in science, engineering, statistics and many more fields. In this project we will learn how derivatives determine the nature and shapes of curves.

3. APPLICATIONS OF DERIVATIVES

3.1 **To find the equation of tangent and normal to a curve at any point**

A tangent is a line grawn to a curve that will meet the curve at a single point and its slope is equivalent to the derivative of the curve at that point. The normal is the line that is perpendicular to the tangent obtained at that point.



The equation of tangent and normal line to a curve of a function can be obtained by the use of derivative. If the curve of a function is given and equation of tangent to a curve at a given point is asked, then by applying the derivative we can obtain the slope and equation of the tangent line. Similarly, we can find the equation of normal line to the curve at that point.Let y=f(x) be the equation of curve, then slope of tangent at any point (x_1, y_1) is given by: $m = \left[\frac{dy}{dx}\right]_{(x_1,y_1)}$.

The normal is perpendicular to the tangent, so the slope of normal at the point is given by : $-\frac{1}{slope\ of\ tangent\ at\ (x_1,y_1)} = -\left[\frac{dx}{dy}\right]_{(x_1,y_1)}$.

Equation of tangent at (x_1, y_1) is given by: $y - y_1 = \left[\frac{dy}{dx}\right]_{(x_1,y_1)} (x - x_1)$

And equation of normal at (x_1, y_1) is given by: $y - y_1 = -\left[\frac{dx}{dy}\right]_{(x_1,y_1)} (x - x_1)$

3.2 To find the interval in which a function is increasing or decreasing

Under this heading we will use applications of derivatives to discover whether a function is increasing or decreasing or none. By the use of derivatives, we can determine if a given function is an increasing or decreasing function. The increasing function is a function that appears to touch the top of x-y plane whereas the decreasing function appears like moving downside corner of x-y plane.

Consider y=f(x) to be a function defined on an interval I contained in the domain of the function f.

f is increasing function in I where$f'(x) \geq 0$

f is strictly increasing in I where $f'(x) > 0$

f is decreasing function in I where $f'(x) \leq 0$

f is strictly decreasing in I where $f'(x) < 0$

3.3 To find the turning point of curve

For a function defined on an interval I, the maxima or minima of function is defined as the values c lying in the interval I which satisfy

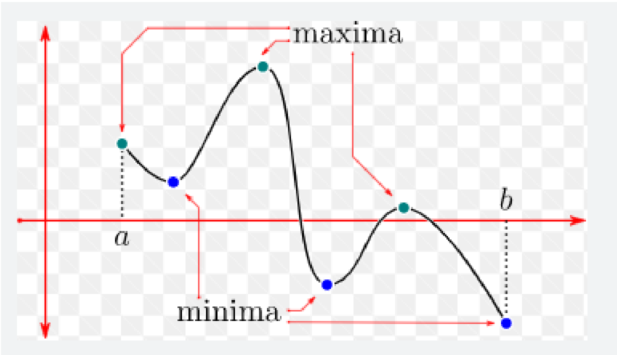
$f(x) \leq f(c) \quad or \quad f(x) \geq f(c) \quad \forall x \in I$

If $f(x) \leq f(c)\ \forall x \in I$, then f has an absolute maximum, and this is also called global maxima.

Similarly, If $f(x) \geq f(c)\ \forall x \in I$, then f has an absolute minimum, and this is also called global minima.

We can also understand the maxima and minima with the help of slope of function:

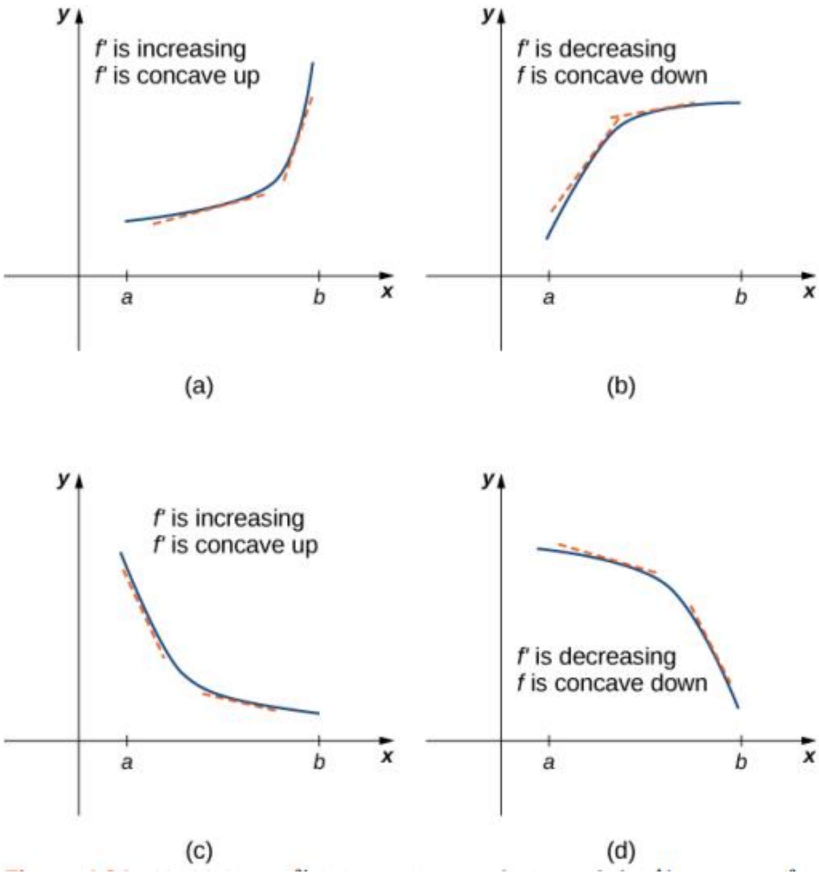
- When the slope of the function changes from +ve to -ve via point c, then c is point of maxima
- When the slope of the function changes from -ve to +ve via point c, then c is point of minima



3.4 To check concavity and points of inflexion

We now know how to determine where a function is increasing or decreasing. However, there is another issue to consider regarding the shape of the graph of a function. If the graph curves, does it curve upward or curve downward? This notion is called the concavity of the function.

Let f be a function that is differentiable over an open interval I. If f ′ is increasing over I, we say f is concave up over I. If f ′ is decreasing over I, we say f is concave down over I.



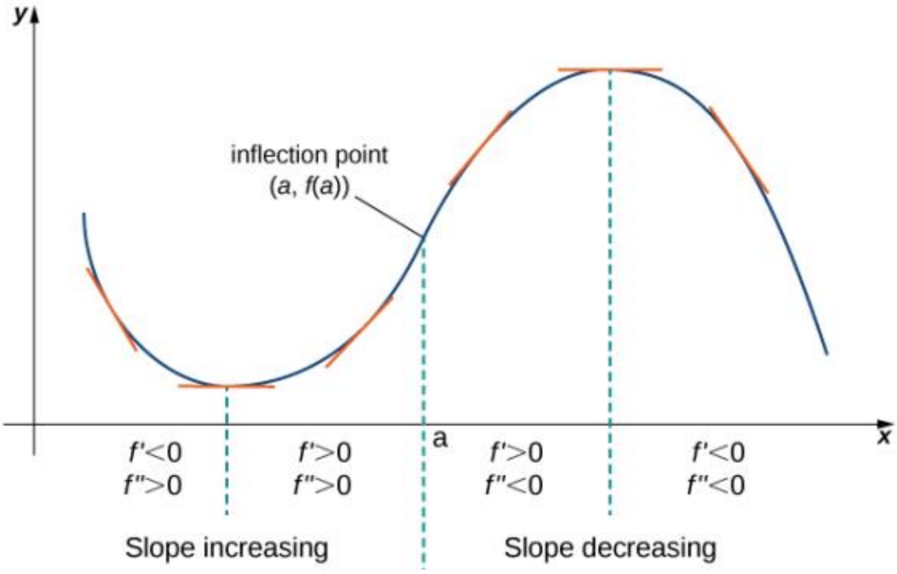
Test for concavity

Let f be a function that is twice differentiable over an interval I.

i. If f ″(x) > 0 for all x ∈ I, then f is concave up over I.

ii. If f ″(x) < 0 for all x ∈ I, then f is concave down over I.

We conclude that we can determine the concavity of a function f by looking at the second derivative of f . In addition, we observe that a function f can switch concavity. However, a continuous function can switch concavity only at a point x if f f″(x) = 0 or f″(x) is undefined. Consequently, to determine the intervals where a function f is concave up and concave down, we look for those values of x where f″(x) = 0 or f ″(x) is undefined. When we have determined these points, we divide the domain of f into smaller intervals and determine the sign of f″ over each of these smaller intervals. If f″ changes sign as we pass through a point x, then f changes concavity. It is important to remember that a function f may not change concavity at a point x even if f ″(x) = 0 or f ″(x) is undefined. If, however, f does change concavity at a point a and f is continuous at a, we say the point (a, f(a)) is an inflection point of f.



4. Conclusion

Through this study, we have now developed the tools we need to determine where a function is increasing and decreasing, as well as acquired an understanding of the basic shape of the graph. By the use of derivatives, we can find the nature of a curve at any point in the domain of function. This study can be applied to many applications in real life.

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