GURU NANAK COLLEGE, SRI MUKTSAR SAHIB

APPLICATIONS OF DIFFERENTIAL EQUATIONS IN BACTERIAL GROWTH

A

PROJECT REPORT

SUBMITTED UNDER

STAR COLLEGE SCHEME

DBT, GOVT. OF INDIA.

Submitted To

Navneet Rana

Assistant Professor,

P.G. Dept. of Mathematics

Submitted By

Prabjot Kaur (B.Sc. III, 5918)

Prabhjot Kaur (B.Sc. III, 5914)

Amandeep Kaur (B.Sc. III, 5929)

Harsharan Kaur (B.Sc. III, 5907)

ACKNOWLEDGEMENT

It is proud of my privilege and pleasure to express my deep sense of gratitude to my project guide, Mrs. Navneet Rana (Assistant Professor, P.G. Department of Mathematics, Guru Nanak College, Sri Muktsar Sahib, Punjab.) for her help, valuable guidance and constant encouragement.

I am also thankful to all teaching staff of P.G. Department of Mathematics who have rendered their invaluable guidance and assistance time to time.

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1. Abstract

Differential Equation applications have significance in both academic and real life. An equation denotes the relation between two quantity or two functions or two variables or set of variables or between two functions. Differential equation denotes the relationship between a function and its derivatives, with some set of formulas. There are many examples, which signify the use of these equations. Life is essentially connected with growth.

In this paper, we are going to discuss the applications of differential equation in bacterial growth. Different phase of bacterial growth curve has been discussed. Then, Mathematical development of growth equation is done. By simplifying the growth equation, a formula is developed which shows that the bacterial growth follows exponential path. All these results are verified numerically also.

2. Introduction

As we know differential equations are important because for many physical systems, one can formulate differential equations that describe how the system changes with time subject to suitable ideal conditions. The derivatives of the function define the rate of change of a function at a point. It is mainly used in fields such as physics, engineering, biology and so on. The primary purpose of the differential equation is the study of solutions that satisfy the equations and the properties of the solutions.

And here in this paper we are going to discuss the applications of differential equation in bacterial growth. The bacterial growth is the complex process which involves many anabolic and catabolic reactions. Life is essentially connected with growth and the process of growth can be studied from different point of view:

a) On the level of the cell or the organism.
b) On the population level.

Microbiology is a study of growth from point of view (b). This is a simple experiment because growth of bacteria can be very fast and requires only small flasks. The growth of microorganism such as bacteria, fungi and algae is of great interest in biology. Laboratory experiments are very expensive and even very difficult to perform. It is impossible to keep the external factor constant in the laboratory. Even there are proper arrangements to do well experiments but still some conditions vary with time. Moreover, errors occur in measuring the population size.

Most information available concerning the growth of microorganism is a result of laboratory studies. There are two approaches to the study of growth under such controlled conditions are batch culture and continuous culture**.** In a batch culture, the growth of single organism or a group is called consortium, and is evaluated using a defined medium to which a fixed amount of food is added at outset. In continuous, there is a steady influx of growth medium and substrate remains the same. Growth under both culture conditions has been described mathematically and well characterized. Here we begin with a review of growth of bacteria under certain conditions and then we discussed how this is related to growth in the environment.

3. Literature review

Typically, to understand and define the growth of a particular microbial isolate, cells are placed in a liquid medium in which the nutrients and environmental conditions are controlled. If the medium supplies all nutrients required for growth and environmental parameters are optimal, the increase in numbers or bacterial mass can be measured as a function of time to obtain a growth curve. Several distinct growth phases [1] can be observed within a growth curve. These include the lag phase, the exponential phase, the stationary phase, and the death phase. Each of these phases represents a distinct period of growth that is associated with typical physiological changes in the cell culture.

The lag phase

It is the first phase observed under the batch conditions in which growth rate must be zero. When an activator is placed into fresh medium, growth begin after a period of time called the lag phase. The lag face is defined to transition to the exponential phase after the initial population has doubled. The lag phase is due to the psychological adaptation of the cell to the culture conditions. In this a time is required for the induction of specific messenger Ribonucleic acid (RNA) and Protein synthesis to meet a new culture requirement. The lag phase may also be due to the low initial density of organism that result in delusion of exoenzymes (enzymes released from cells) and of nutrients that leak from growing cells.

Generally these materials are shared by cells. But these materials are not as easily taken up when cell density is low. So due to low density, initiation of cell growth, division and then transition to exponential phase may be slowed.

The exponential phase

The second phase of growth observed in batch system is the exponential phase. The exponential phase is characterized by a period of exponential growth possible under the conditions present in the batch system. During exponential growth, the rate of increase of cell in the culture is proportional to the number of cells present at any particular time. There are several ways to express this concept both theoretically and mathematically.

The stationary phase

The third face of growth rate is stationary phase. The stationary phase in a batch culture can be defined as a state of no net growth. Although there is no net growth in stationary phase, cells still grow and divide. Growth is simply balanced by an equal number of cells dying. There are several reasons why batch culture may reach stationery phase. One reason is that carbon and energy source or essential nutrients have completely used up. When a carbon is used up, it does not necessarily mean that all growth stops. This is because dying cells can provide a source of nutrients. Growth of dead cells is called endogenous metabolism.

The death phase

The final phase of growth curve is the death phase which is characterized by a net loss of culturable cells. Even in the death phase their maybe individual cells that are metabolising and dividing, but more viable cells are lost than are gained so there is a net loss of viable cells. The death phase often exponential although the rate of death cell is usually slower than the rate during the exponential phase. There are commonly used approaches to measurement of growth because normally the face of most interest to environmental microbiologist is the lag phase, the exponential phase and the time to onset of stationary phase.

In order to get a better understanding of the processes involved, to extend results and to make predictions, mathematical models are indispensable in many areas of science and engineering. For bacterial growth the models usually take the form of differential equations or systems thereof. The coefficients of these equations have traditionally been deterministic, that is they have been assumed to be known and have no variation. Differential equations with deterministic coefficients have been extensively studied, both from the analytical and numerical viewpoints. However, in many situations, equations with random coefficients are better suited in describing the real behaviour of quantities of interest than their counterpart with deterministic coefficients. Randomness in the coefficients may arise because of errors in the observed or measured data, variability in experiment and empirical conditions, uncertainties (variables that cannot be measured, missing data, etc) or plainly because of lack of knowledge.

4. Simple development of a growth equation

Imagine a single cell of a bacterial species in a medium with all nutrients, oxygen, optimal temperature and pH. Assume that cells of this particular species double (divide) under optimal conditions every 3 h (Fig. 1). We thus write for the doubling time (also: generation time) t_d:

$$t_d = 3h$$

To describe the increase of the number of cells over time (growth curve), we have to find an equation that expresses the cell number as a function (f) of time:

cell number = f (time),

meaning that one variable is time (independent variable) and the other variable is the cell number (dependent variable).

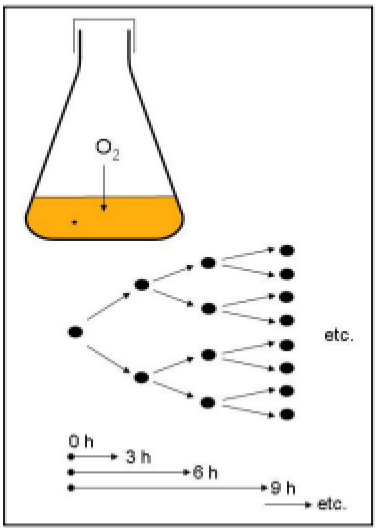

Fig.1: Cell growth that began with a single cell

The following Table.1 presents how the cell number and time correlate:

0 h	3 h	6 h	9 h	The time passed since the beginning of the experiment. For convenience, we here use multiples of the doubling time. Generally: t
1 cell	2 cells	4 cells	8 cells	The number of cells, after each division event (= appearance of a new generation). Generally: N
$=2^0$ cell	$=2^1$ cells	$=2^2$ cells	$=2^3$ cells	The same numbers of cells as above, but written in a manner that reveals the mathematical principle behind the increase of N. Generally: $N=2^n$

However, the recognized equation

$$N = 2^n \tag{1}$$

Using $= \frac{t}{t_d}$, the growth that began with 1 cell can be described as:

$$N = 2^{\frac{t}{t_d}} \tag{2}$$

If growth would begin with any cell number, N_0, the equation would be

$$N = N_0\, 2^{\frac{t}{t_d}} \tag{3}$$

Using $2 = e^{\ln 2}$ and $r = \frac{\ln 2}{t_d}$, the specific growth rate, then the equation (3) becomes

$$N(t) = N_0 e^{rt} \tag{4}$$

5. Mathematical development of the growth equation

During exponential phase, if we let N(t) be the population of a given species at time t, then the simplest model for the growth or decay of the population is that the rate of change is proportional to the size of the population. This is the model proposed by Thomas Malthus in 1798 [2], [3]. The differential equation governing N(t) for this model is

$$\frac{dN}{dt} = rN(t) \tag{5}$$

where r is the growth rate.

Separating variables, we can rewrite the equation as

$$\frac{dN}{N} = r\,dt$$

Integrating both sides,

$$\ln N = rt + C$$

Using initial condition $N(t = 0) = N_0$, we get the value of constant $C = \ln N_0$ and after substituting the value of C, we have

$$\ln N = rt + \ln N_0$$

i.e.

$$\ln \frac{N}{N_0} = rt$$

On simplification, the solution of differential equation is given by

$$N(t) = N_0 e^{rt} \tag{6}$$

Obviously, as the population keeps growing inside the test tubes, there starts to be a competition for the limited resources. In 1838, Verhulst [4] proposed that the growth rate should decrease with the size of the population. This in turn leads to the logistic equation

$$\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N(t) \tag{7}$$

Here r still denotes the growth rate and K is the equilibrium value. The solution, subject to initial value $N(0) = N_0$, is

$$N(t) = \frac{N_0 K}{N_0 + (K - N_0)e^{-rt}} \tag{8}$$

This type of model represents logistic growth model. Further, different models can be developed using effect of a number of nutrients [5].

6. Numerical solution

Assume that the number of bacteria follows an exponential growth model [7]. The initial count in the bacteria culture was 285. The growth rate is 0.05754. Using equation (6), the following table.2 shows the cell count in bacteria culture at different time.

Time (min.)	Cell count (CFU/ml)
20	900
30	1600
40	2847
50	5062
60	9000
70	16000
80	28444

Table.2: Cell count in bacteria culture at different times.

On the basis of these observations, we get the line graph as described in Fig.2. It is easily seen from the graph that the bacterial growth model follow exponential path.

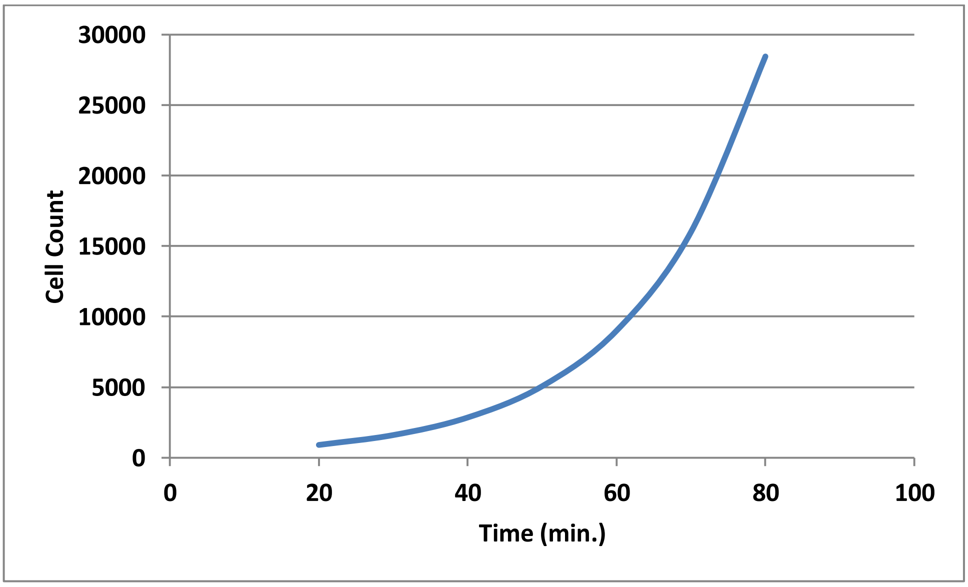

Fig.2: Line graph depicting bacterial growth follow exponential path.

7. Conclusion

Differential equations have very vast applications in real life. A real physical situation can be converted into differential equations using one or more variables. Then, by finding the solution of differential equation, a great knowledge about real physical situation can be obtained. In this way, a bacterial growth model is formed using differential equation.

In this paper, firstly a simple development of growth equation is considered using simple calculations to understand the concept of bacterial growth [6]. After that, the same growth equation is formed mathematically. Numerical solutions are done to verify all theoretical results. It is found out that the bacterial growth follow exponential path.

8. References

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